

Testing the topological nature of the fractional quantum Hall edge

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We carry out numerical diagonalization for much larger systems than before by restricting the fractional quantum Hall (FQH) edge excitations to a basis that is exact for a short-range interaction and very accurate for the Coulomb interaction. This enables us to perform substantial tests of the predicted universality of the edge physics. Our results provide compelling evidence that the behavior of the FQH edge is intrinsically nonuniversal, even in the absence of edge reconstruction, and therefore does not bear a sharp and unique relation to the bulk FQH state.

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The interior of a fractional quantum Hall (FQH) system [1] is gapped, but massless excitations exist at its edge, which constitutes a realization of a one-dimensional electron liquid described generically by the Tomonaga-Luttinger theory [2]. Much attention has been focused on the edge physics since the work of Wen [3], where it was conjectured that the exponent describing the long distance, low energy physics of the chiral (unidirectional) FQH edge is a unique “topological” quantum number for any given FQH state, independent of details, just as the quantum Hall resistance. Our understanding of the ordinary one-dimensional liquids is largely based on the method of bosonization, which exploits a one-to-one correspondence between the fermionic and bosonic Fock spaces in one dimension, and identifies a relationship between the operators on these spaces; specifically, the fermionic field operator $\hat{\psi}(x)$ is related to the bosonic field operator $\hat{\phi}(x)$ through the expression $\hat{\psi}(x) \sim \exp[-i\hat{\phi}(x)]$, which can be established rigorously at the operator level [2]. In the absence of a similar rigorous derivation for the electron field operator at the edge of a FQH system, Wen formulated an effective field theory approach (EFTA) [3] wherein he postulated that the electron operator at the edge of the $1/m$ FQH state, defined by Hall resistance quantization at $R_H = h/(1/m)e^2$, is given by

$$\hat{\psi}(x) \sim e^{-i\sqrt{m}\hat{\phi}(x)}. \quad (1)$$

Antisymmetry under exchange quantizes m to an odd integer value, independent of parameters other than the quantized Hall resistance, which leads to universal properties for the edge physics. A direct test of this assertion is through tunneling of an external electron laterally into the edge of FQH system. For the fractions $\nu = n/(2np + 1)$, a generalization of Eq. (1) predicts the I - V characteristic for electron tunneling into a Fermi liquid to be $I \sim V^\alpha$, where the tunneling exponent has a universal value of $\alpha = 2p + 1$. Ingenious experiments [4–7] have measured the edge exponent by determining the I - V characteristics for tunneling from a three-dimensional Fermi liquid into the FQH edge. While they establish the existence of non-Fermi liquid (Tomonaga-

Luttinger) behavior, with an exponent different from a one-dimensional Fermi liquid ($\alpha = 1$), they also show discrepancy from the EFTA prediction of Wen. Specifically, experiments find an exponent that is continuously varying with the filling factor, and thus is not determined solely by the quantized Hall conductance. Furthermore, the measured exponents are significantly different from the EFTA predictions: at filling factors $\nu = 1/3, 2/5$, and $3/7$, the exponents are $\sim 2.7, 2.3$, and 2.1 , respectively [4–7], to be compared to the EFTA prediction of 3.0 .

A number of theoretical papers have addressed this inconsistency [7–20]. Some of these suggest that the disagreement is due to edge reconstruction, which produces several counter-propagating edge modes (for which the exponent is not universal) [15–18, 21], while some propose that the inconsistency persists even in the absence of edge reconstruction, thus pointing to a more fundamental deficiency of the EFTA [13, 19]. A resolution of this issue is important not only in its own right, but also in view of the potentially useful notion that the character of a bulk FQH state can be ascertained from the behavior of its edge physics [22], which logically rests on the existence of a unique relationship between the two and hence the universality of the latter.

Exact diagonalization studies often provide an unprejudiced, reliable, and decisive tool for testing ideas in the field of the FQH effect. For the edge physics, however, it has not been clear if the discrepancy between the finite system results and the EFTA is intrinsic or a finite size artifact; finite size corrections are more severe for the edge physics [20] because of power law decay of correlations, in contrast to the Gaussian decay in the bulk. Unfortunately, the dimension of the Hilbert space grows exponentially with the number of electrons, making it impossible to increase the system sizes significantly in exact diagonalization studies.

In this Letter we report on substantial microscopic tests of the EFTA by diagonalizing the Coulomb Hamiltonian in a truncated space of edge excitations. Specifically, we consider the edge excitations of the $1/3$ FQH state in the disk geometry, and the truncated space con-

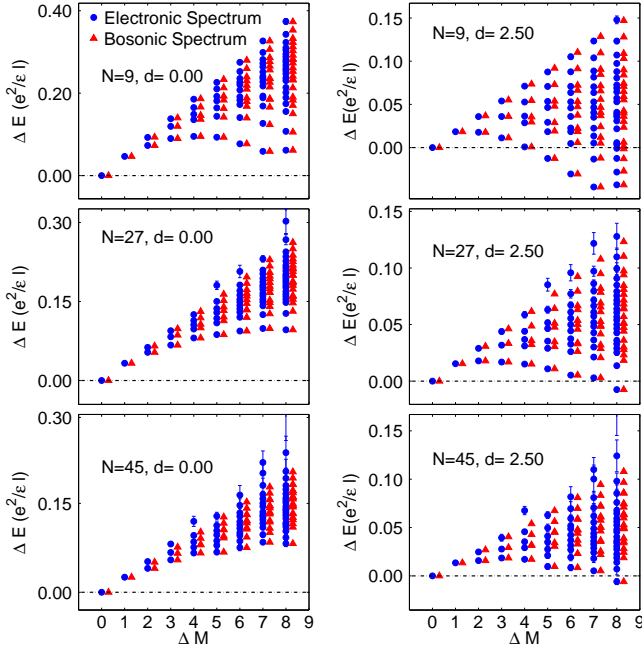


FIG. 1: [Color online] Energy spectrum for the edge excitations of $\nu = 1/3$ for $N = 9, 27, 45$ particles at electron-background separations in the range $d = 0.0$ and 2.5 . Blue dots indicate the energies obtained by CF diagonalization, whereas the adjacent red triangles (shifted along the x axis for clarity) show the bosonic spectra (see text for explanation). All energies are quoted in units of $e^2/\epsilon l$, and measured relative to the energy of the ground state at $\Delta M = 0$. ΔM is the angular momentum of the excited state.

tains all states of the form

$$\Psi_{\alpha}^M = \prod_{j < k} (z_j - z_k)^2 \Phi_{\alpha}^{M^*}, \quad M = M^* + N(N-1), \quad (2)$$

where $z_j = x_j - iy_j$ denotes the electron coordinates as a complex number, M is the total angular momentum of the state, and $\Phi_{\alpha}^{M^*}$ are all lowest Landau level states (labeled by $\alpha = 1, \dots, D^*$) at total angular momentum M^* . The dimension of this basis space is much smaller than the full dimension of the lowest Landau level states at M , which makes it possible to investigate much larger systems; we have studied as many as 45 particles. The restriction to this basis is equivalent to restricting composite fermions [23] to their lowest Λ level (also known as composite fermion Landau level); the space of states can be enlarged in the standard manner [24] by also including at M^* states occupying successively higher Λ levels (and projecting the total wave function onto the lowest Landau level), but that will not be necessary for our present purposes. The lowest- Λ -level approximation for composite fermions is known to be excellent, and we have also confirmed its accuracy explicitly for edge excitations for systems with six and seven particles, for which exact results are available, both with and without the confine-

ment potential. Also, Ψ_{α}^M are the only states that survive if we add to the Coulomb interaction an appropriate infinitely strong short range interaction that annihilates states containing electronic pairs with angular momenta equal to unity; our results below are exact for this model. Therefore, we believe that our truncated Hilbert space ought to capture the topological nature, if it exists, of the edge physics. We believe that this model actually gives the best chance for universal behavior; mixing with higher Λ levels can only spoil it [13].

We consider a system of two dimensional electron gas in disk geometry. The neutralizing background has uniformly distributed positive charge contained in a disk Ω_N of radius $R_N = \sqrt{2N/\nu}$ for a system of N particles at filling factor ν ; the positively charged disk is separated by a distance d from the electron disk (quoted in units of the magnetic length below). The electrons are approximately confined to the same radius because of charge neutrality in the interior. This system is modeled by the following realistic Hamiltonian:

$$H = E_K + V_{ee} + V_{eb} + V_{bb} \quad (3)$$

where the terms on the right hand side represent the kinetic, electron-electron, electron-background, and background-background Coulomb interaction energies, respectively. At large magnetic fields only the lowest Landau level states are occupied, hence the kinetic energy $\hbar\omega_c/2$ (where $\omega_c \equiv eB/m_b c$ is the cyclotron frequency) is a constant and will not be considered explicitly.

The wave functions Ψ_{α}^M are in general not orthogonal, and we use the method of composite-fermion diagonalization (CFD) [24] to orthogonalize them by the Gram-Schmidt procedure, evaluate the Hamiltonian matrix elements, and diagonalize it to obtain the eigenvalues and eigenvectors. All matrix elements and scalar products needed for this purpose are evaluated by the Monte Carlo method, as explained elsewhere in the literature [24]. While sufficiently accurate energy spectrum requires ~ 10 -20 million Monte Carlo iterations, the spectral weights require ~ 200 million iterations for each eigenstate. These numbers do not vary significantly with N , but the computation time increases exponentially with N and ΔM , limiting our study to systems with $N = 45$ for energy, and to $N = 27$ for spectral weights. The energies were calculated for $\Delta M = 1 - 8$ and the spectral weights for $\Delta M = 1 - 4$.

Using the CF diagonalization procedure, we compute the spectra of edge excitations of the $1/3$ state, shown in Fig. (1), for several parameters in the range $N = 6$ -45, $d = 0$ -2.5, and $\Delta M = 0$ -8. Our large system calculations confirm an earlier study [15] that edge reconstruction occurs for d larger than a critical separation, which is approximately 1.5-2.0 magnetic lengths. For d greater than the critical separation, a simple explanation for the observed nonuniversality follows in terms of edge reconstruction; a model that assumes a single chiral

mode is inadequate to describe experiments, forcing one to consider multiple edge modes, which produces nonuniversal results. However, the important question remains whether universality occurs in the absence edge reconstruction.

We address this issue by following the pioneering work of Palacios and MacDonald[9] to test the validity of Eq. (1), upon which the notion of universality rests. Specifically, we compare certain matrix elements of the electron field operator, computed from our CF diagonalization results, with the predictions of the bosonized form in Eq. (1). We also consider d greater than the critical separation for completeness; here, we assume that the ground state remains at $\Delta M = 0$, which can be arranged by adding an ad hoc angular momentum dependent single particle energy term that strongly penalizes the edge excitations responsible for edge reconstruction, but does not change either the eigenfunctions or the energy ordering of states at a given ΔM . (This can be accomplished by adding an appropriate parabolic confinement term which adds to the total energy a term proportional to the total angular momentum.)

The spectral weights are defined by

$$C_{\{n_l\}} = \frac{\langle \{n_l\} | \hat{\psi}^\dagger(\theta) | 0 \rangle}{\langle 0 | \hat{\psi}^\dagger(\theta) | 0 \rangle}, \quad (4)$$

where $\{n_l\}$ represents the bosonic state with occupation $\{n_l\}$, $|0\rangle$ is the vacuum state with zero bosons, and $\hat{\psi}^\dagger(\theta)$ is the electron creation operator at position θ along the edge circle. Here l denotes single boson angular momentum; the total angular momentum is denoted by $\Delta M = \sum_l l n_l$ and the total energy by $\Delta E = \sum_l n_l \epsilon_l$, with ϵ_l being the energy of a single boson at angular momentum l . With the help of $\hat{\psi}^\dagger(\theta) \propto e^{-i\sqrt{m}\phi(\theta)} = \sqrt{z} e^{-i\sqrt{m}\phi_+(\theta)} e^{-i\sqrt{m}\phi_-(\theta)}$, $\hat{\phi}_+(\theta) = -\sum_{l>0} (1/\sqrt{l}) a_l^\dagger e^{il\theta} = \hat{\phi}_-(\theta)$, it is straightforward to obtain the predictions for the spectral weights:

$$|C_{\{n_l\}}|^2 = \frac{m^{n_1+n_2+\dots}}{n_1! n_2! \dots 1^{n_1} 2^{n_2} \dots} \quad (5)$$

We note that the denominator in Eq. (4) eliminates the unknown normalization constant \sqrt{z} .

In order to obtain the spectral weights from our electronic spectra, it is natural to identify the vacuum state $|0\rangle$ with the ground state of interacting electrons at $\nu = 1/m$, denoted by $|\Psi_0^N\rangle$, and the field operator has the standard meaning of $\hat{\psi}^\dagger(\theta) = \sum_l \eta_l^*(\theta) c_l^\dagger \equiv \sum_l \psi_l^\dagger(\theta)$, where c_l^\dagger and c_l are creation and annihilation operators for an electron in the angular momentum l state. The denominator of Eq. (4) corresponds to

$$\langle 0 | \hat{\psi}^\dagger(\theta) | 0 \rangle = \frac{\langle \Psi_0^{N+1} | \hat{\psi}_{L_0}^\dagger(\theta) | \Psi_0^N \rangle}{\sqrt{\langle \Psi_0^{N+1} | \Psi_0^{N+1} \rangle \langle \Psi_0^N | \hat{\psi}_{L_0}(\theta) \hat{\psi}_{L_0}^\dagger(\theta) | \Psi_0^N \rangle}}, \quad (6)$$

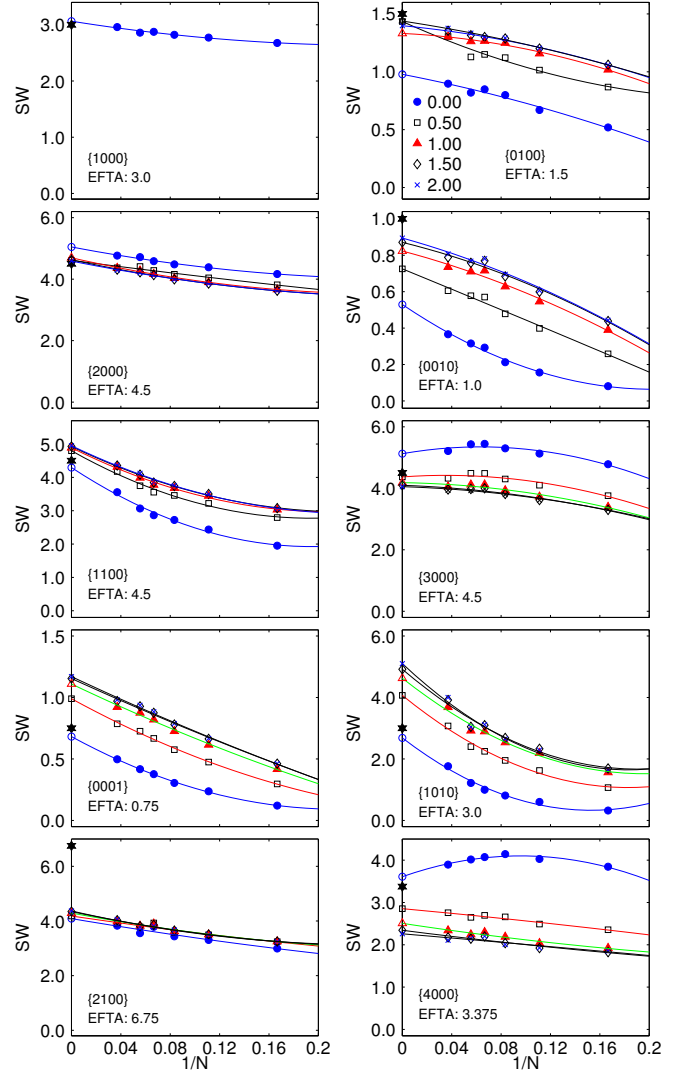


FIG. 2: [Color online] N dependence of spectral weights $|C_{\{n_l\}}|^2$ for several states $\{n_l\}$, indicated on each panel, and various separations d (quoted in units of the magnetic length in the $\{0100\}$ panel). The EFTA prediction from Eq. (1) is indicated by a star on the y-axis, with the value also given on each panel. The points on the y-axis are determined by a quadratic fit to the finite N results.

where $|\Psi_0^N\rangle$ is the ground state of N interacting electrons at $\nu = 1/m$, and $L_0 = mN$. The numerator is similarly defined as as

$$\langle \{n_l\} | \hat{\psi}^\dagger(\theta) | 0 \rangle = \frac{\langle \Psi_{\{n_l\}}^{N+1} | \hat{\psi}_L^\dagger(\theta) | \Psi_0^N \rangle}{\sqrt{\langle \Psi_{\{n_l\}}^{N+1} | \Psi_{\{n_l\}}^{N+1} \rangle \langle \Psi_0^N | \hat{\psi}_L(\theta) \hat{\psi}_L^\dagger(\theta) | \Psi_0^N \rangle}}. \quad (7)$$

Here, we have

$$\hat{\psi}_L^\dagger | \Psi_0^N \rangle = \mathcal{N}_L \mathcal{A} \left[z_{N+1}^L e^{-|z_{N+1}|^2/4} \Psi_0^N(z_1, z_2, \dots, z_N) \right], \quad (8)$$

where \mathcal{A} is the antisymmetrization operator, \mathcal{N}_L is the

normalization constant, and $L \equiv L_0 + \Delta M$ is the angular momentum of added electron.

The wave function $\Psi_{\{n_l\}}^{N+1}$, the electronic counterpart of the bosonic state $|\{n_l\}\rangle$, clearly represents an excited state at total angular momentum $M = \Delta M + mN(N+1)/2$, which should also be related to the total angular momentum of the N particle ground state through $M = L + mN(N-1)/2$. At each ΔM , there are in general many eigenstates. Following Ref. [15] we identify ϵ_l , the energy of a single boson with angular momentum l , with the lowest energy at $l = \Delta M$ in the calculated spectrum. Using the equations $\sum_l l n_l = \Delta M$ and $E_{\{n_l\}} = \sum_l n_l \epsilon_l$, the energies of the all bosonic states $\{n_l\}$ can now be obtained (see Fig. 1), which can then be identified with the corresponding electronic states. We note that in the lowest Λ level subspace, the numbers of electronic and bosonic states are equal at each ΔM , so a one to one correspondence between the two sets of states can be established from their energy ordering. For small systems (for example, $N = 9$ in Fig. 1), the CFD spectra and the bosonic spectra are very close to each other, which explicitly confirms the interpretation of the lowest branch as the single boson branch. The agreement between the electronic and bosonic spectra becomes less accurate with increasing N or ΔM , but still remains adequate for the low energy states, which will be our focus. (The higher energy states of the spectra shown in Fig. 1 mix with higher Λ level excitations of composite fermions, not considered in our model.)

Figure (2) shows the squared spectral weights for different excited states as a function of N and d . A quadratic fit extrapolates the result to the thermodynamic limit $1/N = 0$. The EFTA predictions from Eq. (1) are also shown in each panel. These plots demonstrate the central result of our work: the spectral weights are nonuniversal; they depend on d ; and they do not extrapolate to the EFTA value. For the $\{1000\}$ excitation the thermodynamic result agrees with predicted result of 3.0 for all d ; however, in this case our truncated space contains a single (center-of-mass) excitation, the wave function for which is independent of interactions (within our model), and therefore the agreement is not meaningful. For many cases, the deviation from the EFTA value is substantial; even the spectral weights of single boson states, such as $\{0100\}$ and $\{0010\}$ exhibit significant d dependence.

Our study thus indicates that Eq. 1 is not valid for the $1/m$ FQH edge for the Coulomb interaction, and therefore there is no reason to expect the edge exponent to be a topological quantum number for the $1/m$ FQH state; this conclusion very likely holds for other FQH states as well, given that their edges are believed to be more complex. The problem of how in reality the electron field is related to the bosonic field remains unresolved, however. Following Ref. [19] one may abandon the antisymmetry requirement and try an expression of the type

$\hat{\psi}(x) \sim e^{-i\sqrt{\alpha}\hat{\phi}(x)}$ with arbitrary α ; we have found that no single value of α gives a satisfactory description of all spectral weights that we obtain from numerical diagonalization. It is also worthwhile here to mention the possibility of a nonlocal relation between the two, as suggested in Ref. [13].

Other models for edge confinement have been used. One such model [15] restricts the single particle angular momentum to a maximum value of $l_{\max} = 3(N-1) + l_0$, which may be a reasonable approximation for cleaved edge overgrowth [7]. However, angular momentum conservation shows (and our Monte Carlo calculations explicitly confirm) that in this model the actual spectral functions identically vanish for $\Delta M > l_0$, resulting in an even more substantial disagreement with the EFTA predictions.

In summary, our study allows an estimate of certain thermodynamic properties of the FQH edge for a realistic Coulomb model, and makes what we believe to be a compelling case that the behavior at the FQH edge is intrinsically nontopological, as also demonstrated by experiments. This result has obvious implications for the program of determining the nature of a bulk FQH state by probing its edge.

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